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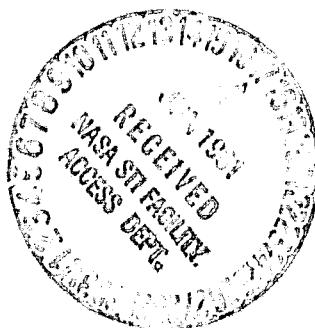
# RELATIONSHIPS BETWEEN DIFFUSE REFLECTANCE AND VEGETATION CANOPY VARIABLES BASED ON THE RADIATIVE TRANSFER THEORY

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VARIABLES BASED ON THE RADIATIVE TRANSFER THEORY\*

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## ABSTRACT

Useful relationships were formulated to describe variations of the diffuse spectral reflectance in terms of vegetation canopy variables, such as biomass. The relationships were based on the solution of the two-stream approximation of the radiative transfer equation. Out of the lengthy original expression of the diffuse reflectance formula, simple working equations were derived by employing characteristic parameters, which are independent of the canopy coverage and identifiable by field observations. The typical asymptotic nature of reflectance data that is usually observed in biomass studies was clearly explained. The usefulness of the simplified equations was demonstrated by the exceptionally close fit of the theoretical curves to two separately acquired data sets for alfalfa and shortgrass prairie canopies.

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## INTRODUCTION

Agricultural remote sensing research in recent years has been largely concerned with developing a fundamental quantitative understanding of the relationships between spectral responses and vegetation scene factors and how they are related to important agronomic parameters, such as plant biomass and crop yield. Such quantitative relationships are essential for extracting useful information from remotely sensed data for applications, such as forage management and pre-harvest prediction of crop yield.

Numerous empirical relationships have been proposed for crop canopy assessments. Most of these remote sensing techniques rely on red and near-infrared reflectance or radiance ratios, which have been summarized by Tucker (1979). There is considerable evidence for a variety of cover types that red and photographic infrared spectral data are highly sensitive to the projected green leaf area index or green leaf biomass (Deering, 1978; Tucker, 1979; Holben et al., 1980). Additionally, similar techniques have been found useful for indirectly assessing drought stress (Thompson and Wehmanen, 1979) and evapotranspiration (Wiegand et al., 1979).

Limitations and inconsistencies in the spectral relationships among the various cover types and conditions have prevailed, however, due to their lack of a theoretical foundation. Tucker (1980) concluded that their utility in assessing standing crop biomass is tied to the relationship of the green leaf area index to the standing crop biomass for the cover type in question, and thus these relationships to standing crop biomass are not temporally consistent. Failure to take spectral measurements near solar noon results in additional inconsistencies in empirical models due to solar zenith angle effects (Duggin, 1977; Kriebel, 1978; Kimes et al., 1980).

This investigation was directed toward developing a basic relationship between vegetation canopy variables and diffuse spectral reflectance based on the radiative transfer theory. The Kubelka-Munk model (Allen and Richardson, 1968) was extended to account for anisotropic diffusion of light within the canopy. The goal of this study was to establish a practical procedure for analysis of biomass/reflectance data.

#### MODIFICATION OF KUBELKA-MUNK MODEL

Variations of the monochromatic diffuse radiation within vegetative canopies have been described by means of a two-parameter concept involving coefficients of absorption and scattering (Allen and Richardson, 1968). This treatment is known as the Kubelka-Munk (KM) theory, which is applicable to homogeneous, perfectly diffusing media with light-absorbing and light-scattering elements. Such a treatment is seldom exact, but light intensity passing half-transparent materials has often been well approximated by the theory (Wendlandt and Hecht, 1956; Kortüm, 1969).

Most vegetative canopies consist of several distinct components that result in anisotropic canopy reflectance. Examples are the reflectance differences between upper and lower surfaces of many plant leaves (Gates et al., 1965) and bidirectional scattering effects of individual leaves (Breece and Holmes, 1971). Another example is inhomogeneous distributions of leaf orientation. The traditional two-parameter representation of the radiative transfer equation is certainly inadequate to take into account such phenomena. For this reason, the KM equations were extended by employing the two different sets of absorption and scattering coefficients as follows:

$$-\frac{dE_-}{d(\rho z)} = - (a_- + \gamma_-) E_- + \gamma_+ E_+ \quad (1)$$

$$\frac{dE_+}{d(\rho z)} = - (a_+ + \gamma_+) E_+ + \gamma_- E_- \quad (2)$$

where

- $E_-$  = monochromatic light intensity in the downward direction
- $E_+$  = monochromatic light intensity in the upward direction
- $z$  = distance from the canopy top (negative in the downward direction)
- $a_-$  = absorption coefficient associated with  $E_-$
- $a_+$  = absorption coefficient associated with  $E_+$
- $\gamma_-$  = scattering coefficient associated with  $E_-$
- $\gamma_+$  = scattering coefficient associated with  $E_+$
- $\rho$  = density of plant canopy variable (e.g., biomass per unit volume).

Here,  $d(\rho z)$  is the differential of biomass (biomass per unit area in a vertical distance segment  $dz$ ). The variable " $\rho z$ " can be converted into other canopy variables, such as leaf area index. The choice of the appropriate parameter is often a matter of convenience in describing relationships between measured quantities. However, biomass is preferable to many other parameters, since the volume scattering by randomly oriented leaf elements is more appropriate for most plant canopies than multiple scattering in stratified leaf layers. The notations are simplified by omitting subscripts for the wavelength dependence of the variables and parameters.

In this formulation the backward scattering of only the diffuse light is taken into account so that the problem remains one-dimensional (Fig. 1). It is assumed that there are negligible

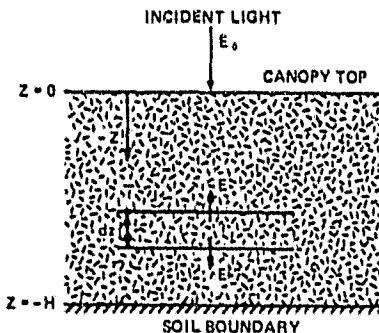


Fig. 1. One-dimensional Plant Canopy Model

contributions due to the emissivity of any substances in the wavelength considered. Other underlying assumptions are that the coefficients of bulk absorption and scattering are greater than zero. These assumptions are needed to avoid the problem of ambiguity, which occurs in cases where any coefficients vanish or become negative. The set of the present governing equations is a modification of the KM model, in which  $a_- = a_+$  and  $\gamma_- = \gamma_+$ .

The general solution set to the coupled differential equations is given by

$$E_- = A_1 e^{m_1 \rho z} + A_2 e^{m_2 \rho z} \quad (3)$$

$$E_+ = B_1 e^{m_1 \rho z} + B_2 e^{m_2 \rho z} \quad (4)$$

where

$$m_1 = D+K \quad (5)$$

$$m_2 = D-K \quad (6)$$

$$D = (a_- - a_+ + \gamma_- - \gamma_+)/2 \quad (7)$$

$$K = (D^2 + a_- a_+ + \gamma_- \gamma_+ + a_+ \gamma_-)^{1/2} \quad (8)$$

and  $A_1$ ,  $A_2$ ,  $B_1$  and  $B_2$  are constants to be determined by boundary conditions. It can be shown without difficulty that

$$m_1 > 0$$

$$m_2 \leq 0$$

and also that, if  $a_- = a_+$  and  $\gamma_- = \gamma_+$ ,

$$D = 0$$

$$m_2 = -m_1.$$

Convention shows in this type of problem that a common boundary condition is

$$E_- = E_0 \text{ at } z = 0, \quad (\text{B.C. 1})$$

where  $E_0$  is the intensity of the incident monochromatic light.

#### *Large Biomass Canopy Solution*

An important physical insight to the problem can be learned by examining the following hypothetical boundary condition:

$$E_- \text{ and } E_+ : \text{ finite at } z = -\infty \quad (B.C. 2)$$

when the canopy height or biomass is infinitely large. This hypothetical case yields the solution set given by

$$E_- = E_0 e^{m_1 \rho z} \quad (9)$$

$$E_+ = R_v E_- \quad (10)$$

where

$$R_v = S_N - K_N \quad (11)$$

$$S_N = (a_- + a_+ + \gamma_- + \gamma_+)/2 \gamma_+ \quad (12)$$

$$K_N = K/\gamma_+ \quad (13)$$

and  $z \leq 0$ . In the present expression,  $R_v$  is the diffuse reflectance of the infinitely tall (or dense) canopy, which is a hypothetical situation. It is the reflectance of the vegetation canopy by which the influence of the background reflectance is eliminated. The so-called KM relationship for diffuse reflectance (Park, 1980):

$$R_v = 1 + \frac{a}{\gamma} - \sqrt{\left(\frac{a}{\gamma}\right)^2 + 2 \frac{a}{\gamma}} \quad (14a)$$

$$\frac{a}{\gamma} = \frac{(1 - R_v)^2}{2 R_v} \quad (14b)$$

is the special case of Eq. 11 where  $a = a_- = a_+$  and  $\gamma = \gamma_- = \gamma_+$ . In Eqs. 11 and 14a it is important to note that the diffuse reflectance is a function of ratio parameters,  $a_-/\gamma_+$ ,  $a_+/\gamma_+$  and  $\gamma_-/\gamma_+$  or  $a/\gamma$ . In Eq. 9,  $m_1$  is the mass attenuation coefficient of light, and the product  $m_1 \rho$  is the reciprocal of the penetration depth when the canopy is infinitely tall or dense. Eq. 9 is the typical form of the Beer-Lambert law (Wendlandt and Hecht, 1966).

### Model Solution

The background effects cannot be excluded in most canopy reflectance measurements. To account for the soil background effects in the model of canopy height  $H$ , the second

boundary condition is substituted by

$$E_+ = R_b E_- \text{ at } z = -H \quad (\text{B.C. 3})$$

where  $R_b$  is the reflectance of the background surface under the canopy. The solution to Eqs. 1 and 2 satisfying Boundary Conditions 1 and 3 is given by

$$E_- = \frac{E_0}{C} \left\{ \left( \frac{r}{R_v} - R_b \right) e^{m_1 \rho(H+z)} - (R_v - R_b) e^{m_2 \rho(H+z)} \right\} \quad (15a)$$

$$= \frac{E_0}{C} \left\{ (S_N + K_N - R_b) e^{m_1 \rho(H+z)} - (S_N - K_N - R_b) e^{m_2 \rho(H+z)} \right\} \quad (15b)$$

$$E_+ = \frac{E_0}{C} \left\{ \left( \frac{r}{R_v} - R_b \right) R_v e^{m_1 \rho(H+z)} - (R_v - R_b) \frac{r}{R_v} e^{m_2 \rho(H+z)} \right\} \quad (16a)$$

$$= \frac{E_0}{C} \left\{ (S_N + K_N - R_b) (S_N - K_N) e^{m_1 \rho(H+z)} - (S_N - K_N - R_b) (S_N + K_N) e^{m_2 \rho(H+z)} \right\} \quad (16b)$$

where

$$C = \left( \frac{r}{R_v} - R_b \right) e^{m_1 \rho H} - (R_v - R_b) e^{m_2 \rho H} \quad (17a)$$

$$= (S_N + K_N - R_b) e^{m_1 \rho H} - (S_N - K_N - R_b) e^{m_2 \rho H} \quad (17b)$$

$$r = \gamma_- / \gamma_+ \quad (18a)$$

$$= S_N^2 - K_N^2 \quad (18b)$$

$$= R_v (S_N + K_N) \quad (18c)$$

and  $-H \leq z \leq 0$ . It should be noted that considerable caution is needed when the relationships are applied to field data, because radiance within canopies can vary drastically from place to place due to the random nature of the leaf spacing. The use of the solutions can be justified if the average radiances are observed over large areas. In remote sensing applications of canopy reflectance models, reflected radiation is measured far above the canopy by airborne or satellite sensors, and thus more adequately represent the values of the canopy reflectance for the area viewed by the sensor.

## REFLECTANCE/BIOMASS RELATIONSHIPS

The ability to monitor vegetation canopies using remote sensing techniques results from inherent or environmentally induced reflectance differences among plants and plant types during their growth cycle. In this section the reflectance/biomass relationships will be derived from the solution to the extended KM equations and their important properties will be discussed.

The apparent diffuse reflectance formula for a vegetative canopy is obtained by letting  $z=0$  from Eq. 16 as

$$\begin{aligned} R &= E_+ (z=0)/E_0 \\ &= \frac{(\frac{r}{R_v} - R_b) R_v - (R_v - R_b) \frac{r}{R_v} e^{-2K\rho H}}{\frac{r}{R_v} - R_b - (R_v - R_b) e^{-2K\rho H}} \quad (19a) \end{aligned}$$

$$= \frac{(S_N + K_N - R_b)(S_N - K_N) - (S_N - K_N - R_b)(S_N + K_N) e^{-2K\rho H}}{S_N + K_N - R_b - (S_N - K_N - R_b) e^{-2K\rho H}} \quad (19b)$$

$$\text{or } e^{-2K\rho H} = \frac{(r - R_b R_v)(R_v - R)}{(r - R R_v)(R_v - R_b)} \quad (19c)$$

$$= \frac{(S_N + K_N - R_b)(S_N - K_N - R)}{(S_N + K_N - R)(S_N - K_N - R_b)} \quad (19d)$$

if  $R_v \neq R_b$ . This is the most general formula for the apparent canopy reflectance. It should be noted that the constant parameter  $D$  does not appear in this relationship; thus its expression is considerably less complex. These exponential equations show asymptotic behaviors as the canopy height  $H$  (or biomass  $\rho H$ ) approaches the two extreme values:

and 
$$\begin{aligned} R &\rightarrow R_v & \text{as} & \quad H \rightarrow \infty \quad (\text{or } \rho H \rightarrow \infty) \\ R &\rightarrow R_b & \text{as} & \quad H \rightarrow 0 \quad (\text{or } \rho H \rightarrow 0). \end{aligned}$$

The asymptotic properties agree with most observations of canopy reflectance (Pearson, 1973; Tucker, 1977; Deering, 1978).

### *Relationships Simplified*

It is highly desirable to simplify complicated formulas for easier use and wider applicability. A couple of simplified relationships can be derived from Eq. 19. The general formula can be approximated by the simple exponential equation:

$$R \approx R_v - (R_v - R_b) e^{-2K\rho H} \quad (20)$$

when  $r$  is sufficiently larger than one, since  $0 < R_v, R, R_b < 1$ . Deviation of the apparent reflectance  $R$  by this equation from that by Eq. 19 is given approximately by

$(R_b - R_v)^2 R_v \exp(-2K\rho H)/r$ . Even if  $r$  is close to one, the approximation by Eq. 20 may be sufficient for practical use, since  $(R_b - R_v)^2 R_v \exp(-2K\rho H) \ll 1$  in most cases. In field observations the uncertainty of reflectance data is considerably large due to the presence of the direct solar radiation (Kriebel, 1976). Hence, such an approximation might be justified in analysis of the biomass/reflectance data when bidirectional reflectance observations are used as the diffuse reflectance data. In the equation the exponential term is the contribution of the background reflectance to the apparent canopy reflectance. It shows that the soil background effects will be significant when the difference between  $R_v$  and  $R_b$  is large and biomass  $\rho H$  (or canopy height  $H$ ) is small.

The asymptotic properties of the relationship are clearly seen at the two extreme values of the canopy biomass in Eq. 20. The equation also indicates the ranges of the apparent canopy reflectance as

$$R_v \leq R \leq R_b \quad \text{if} \quad 0 < R_v \leq R_b < 1,$$

and

$$R_v \geq R \geq R_b \quad \text{if} \quad 1 > R_v \geq R_b > 0.$$

This working equation is simple in form and similar to many biological formulas possessing various asymptotic properties. For example, a similar relation has been employed to empirically fit spectroreflectance and chlorophyll data (Pearson, 1973; Tucker, 1977). Such a simplicity would enhance the utility and acceptance of the relationship in agricultural applications.

Another commonly cited case is that  $a_- = a_+$  and  $\gamma_- = \gamma_+$ . In this case Eq. 19c becomes

$$e^{-2K\rho H} = \frac{(1 - R_b R_v)(R_v - R)}{(1 - R R_v)(R_v - R_b)} \quad (21)$$

since  $r = \gamma_-/\gamma_+ = 1$ . A relationship equivalent to Eq. 21 has been derived by Allen and Richardson (1968), who employed the leaf area index (LAI) instead of canopy biomass  $\rho H$ . The relationship, which has been used to predict canopy LAI as a function of the apparent canopy reflectance at  $0.4 \mu\text{m}$ , was depicted for shortgrass prairie vegetation (*Bouteloua gracilis*) and its asymptotic property was also cited without elaboration by Oliver and Smith (1973).

#### *Characteristic Parameters*

The values of  $R$ ,  $R_b$ ,  $R_v$  and  $\rho H$  may be measured to some degree and, then the parameter  $K$  may be computed. However, a series of observations of apparent reflectance  $R$  and other canopy variables, such as  $H$  or  $\rho H$ , can lead to estimation of the other parameters:  $R_b$ ,  $R_v$  and  $K$  or  $K\rho$ . Once the values of the characteristic parameters are known for a growth cycle of vegetation, the plant biomass can be assessed nondestructively by observing apparent canopy reflectances. It is possible to establish reflectance/biomass relationship curves for different crops or vegetation types by finding proper values of these characteristic parameters ( $R_b$ ,  $R_v$  and  $K$  or  $K\rho$ , as well as  $r$ ) for various agronomic and environmental conditions. The value of  $r$  yields the first clue for the anisotropic property in canopy reflectance characteristics.

#### *Model Evaluation*

The canopy reflectance relationships, Eqs. 19 through 21, were tested for the biomass/reflectance data of 1) alfalfa and 2) shortgrass prairie vegetation. The alfalfa data were taken from seven experimental plots having different plant density. The different biomass levels were created through selective thinning within small plots ( $3\text{m}^2$ ), which were contained within a larger, uniform stand of alfalfa. The canopy was approximately 45 - 50 cm

high, contained very little brown plant material, and was ready for the fall hay cutting.

Alfalfa canopy spectral reflectances were acquired in a sampling mode (10 samples per small plot) using a two-channel, red and photographic infrared portable radiometer with spectral bands centered at about .68 and .80  $\mu\text{m}$ , respectively. The observations were made under various sky and illumination conditions on three consecutive days (Table 1).

Table 1. Reflectance of Alfalfa Canopies at Wavelength .68 $\mu\text{m}$   
Observed on October 11-13, 1978

I.D.	OBSERVATION DAY/TIME	DRY BIOMASS (kg/ha)							PLOT ILLUMINATION CONDITION	NOMINAL SUN ELEVATION (deg.)
		0	70	960	1,650	2,280	3,660	3,850		
A1	11/ 9:13- 9:37	.174	.166	.084	.042	.034	.032	.028	Sunny	32
A2	9:47-10:00	.186	.172	.084	.041	.035	.028	.030	"	36
A3	10:22-10:34	.180	.179	.089	.047	.036	.029	.027	"	40
A4	11:05-11:19	.189	.169	.091	.050	.033	.028	.027	"	43
A5	12:00-12:14	.186	.172	.090	.047	.031	.027	.027	"	44
A6	13:05-13:21	.185	.168	.075	.051	.030	.025	.025	"	40
A7	14:01-14:12	.176	.162	.076	.058	.028	.026	.022	"	34
A8	14:56-15:07	.175	.159	.070	.059	.033	.025	.026	"	27
A9	15:30-15:43	.189	.164	.073	.067	.030	.026	.024	"	21
B1	12/13:50-14:24	.178	.160	.075	.047	.032	.025	.026	Hazy	34
B2	14:41-14:56	.183	.160	.078	.059	.035	.030	.029	"	28
C	13/12:31-13:03	.235	.212	.108	.081	.047	.033	.035	Hazy with high cirrus	41
D	13/12:33-13:04	.270	.255	.115	.166	.051	.042	.038	Hazy with high cirrus and shaded	41

Under the clear sky conditions, measurements were made at nine different sample times during the day to examine the diurnal/sun elevation effects. Measurements were also made under more diffuse lighting conditions of hazy skies and skies containing high thin cirrus clouds. One data set was also collected for a no-direct-sunlight condition by artificially shading the plant canopy.

As the model was developed for diffuse illumination conditions, the reflectance data for the overcast day were potentially the most favorable for analysis of the model relationships. The clear sky conditions were considered to be a crucial test of whether the relationships were applicable to the anisotropic illumination, which is a deviation from the diffuse irradiance assumption.

The biomass/reflectance data of the shortgrass prairie vegetation, dominated by blue grama grass (*Bouteloua gracilis*), were chosen from the field data of 27 native grassland plots, as reported by Pearson and Miller (1972). Their spectroreflectances were measured, using a narrow band spectrometer of similar design as that used in the alfalfa field study. The corresponding biomass data were those of the total (green + dead) dry standing biomass clipped from each 1/4 square meter plot within the field-of-view of the spectrometer.

Suitability of the model relationship to the reflectance/biomass data depends mostly on the parameter values. All the physics of canopy reflectance characteristics is depicted in terms of these bulk parameters of the model. Spectral reflectance  $R_b$  of the background surface can be measured without difficulty if the areas of negligible canopy can be found in the study sites. Vegetation reflectance  $R_v$  might be approximated by extrapolation of data points when observations cover a wide range of canopy development stages or when observed data have "leveled off." However, the parameter  $K$  (or the product  $K\rho$ ) is the resultant attenuation coefficient of the canopy as a half transparent medium, and its value can be obtained indirectly by evaluating Eq. 15, 16, 19 or 20 based on a series of observed data. For example, if all the other constant parameters ( $R_b$  and  $R_v$ ) are known, only one set of  $R$  and  $\rho H$  data will be enough to determine  $K$  by Eq. 20. In the event that two or more available data sets may yield different values of  $K$ , the optimal solution for  $K$  is desired. Such an optimum estimate of an unknown parameter set can generally be found as a solution to the observation equations generated by a working equation for given data sets.

In this investigation  $R_b$ ,  $R_v$ , and  $K$  were estimated such that their values yielded the best approximation of Eq. 19, 20 or 21 for given data set, that is,

$$\text{Min } \sum_i W_i \{ f_i (R_b, R_v, K; r; B_i, R_i) \}^2,$$

where

$$B_i = (\rho H)_i : \text{Canopy biomass of the } i\text{-th obs.} \quad (22)$$

$$f_i = \begin{cases} 1 - \frac{(r - R_i R_v) (R_v - R_b)}{(r - R_b R_v) (R_v - R_i)} e^{-2KB_i}, & (23) \\ 1 - \frac{(1 - R_i R_v) (R_v - R_b)}{(1 - R_b R_v) (R_v - R_i)} e^{-2KB_i}, & (24) \\ 1 - \frac{R_v - R_b}{R_v - R_i} e^{-2KB_i}, & (25) \end{cases}$$

$w_i$  = weight for the  $i$ -th observation,

and  $R_i$  is the observed spectral reflectance for the canopy of biomass  $B_i$ . Eqs. 23, 24, and 25 are equivalent to Eqs. 19, 21, and 20, respectively, if  $f_i = 0$ . The best set of the parameter estimates minimizes the weighted sum of the squared  $f_i$  for the whole data and was computed using the IMSL Subroutine ZXSSQ (IMSL Library, 1979). When the solution was searched by the subroutine, Eq. 23 was inefficient for computation and, hence, not used in the later analysis.

#### *Model and Observed Relationships*

The estimates of the characteristic parameters obtained by Eq. 24 or 25 (Table 2) produced curves clear depicting the observed relations between biomass and spectral reflectance (Figs. 2 and 3). No noticeable differences were found between the results from Eqs. 20 and 21 (Table 2). For shortgrass prairie canopy data of Pearson and Miller no definitive best relationship could be drawn due to the large scatter of the data. In the optimum model solution the reflectance  $R_v$  of shortgrass prairie infinite canopy condition (i.e., that of the sufficiently large biomass as seen in the asymptotic character) was zero, which was out of the acceptable range for the solution, but it was certainly more realistic than the negative reflectance shown in the empirical linear regression relationship (Fig. 3), which was achieved at only 4200 kg/ha. The exponential formula, Eq. 20, worked favorably for analysis of the biomass/reflectance data – deviating by about 2.4% for biomass estimates and less than 0.2% for apparent reflectance estimates from the more complicated formula.

Table 2. Estimates of Reflectance Characteristic Parameters and Comparison of the Two Results Obtained by Eqs. 20 and 21

	K (10 <sup>-6</sup> ha. kg)	R <sub>s</sub>	R <sub>b</sub>	R <sub>s</sub> / R <sub>b</sub>	a γ	AVE. DIFFERENCE BETWEEN TWO EST.		WEIGHT OPTION
						BIOMASS (%)	REFL. (%)	
A1	617	027	169	138	17.7	1.7	0.6	1
A2	608	026	172	154	18.0	4.7	0.7	1
A3	552	025	181	137	19.1	2	0.8	1
A4	629	026	191	135	18.4	2	0.8	1
A5	741	026	198	133	18.0	2	0.7	1
A6	680	024	190	126	19.9	2	0.7	1
A7	537	019	168	115	20.9	2.1	0.7	1
A8	515	022	165	133	21.8	3.1	0.7	1
A9	584	022	178	125	21.5	3.3	0.7	1
B1	568	023	171	134	20.9	2.1	0.7	1
B2	604	028	179	135	17.1	1.6	0.7	1
C	444	026	224	116	18.3	4.9	13	1
D	486	013	258	127	14.3	6.9	17	1
Ave.	582	025	188	134	18.9	2.4	0.83	-
C.V.	14	12	14	10	11	88	36	-
PM*	166	0	186	0	∞	0	0	2

NOTE: Weight option 1 - No weight was used

Weight option 2 - A normalized distance from each data set to the mean was used for the weight

C.V. - Coefficient of variation

\* Results for shortgrass prairie canopy data of Pearson and Miller (1972)

See the text for the other symbols and discussions

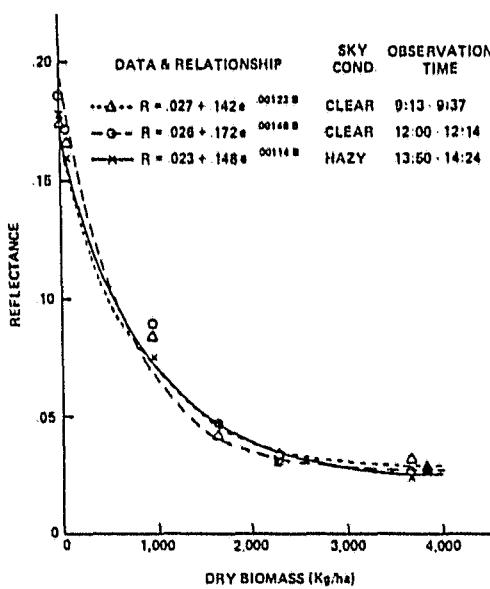


Fig. 2. Spectral Reflectance vs. Dry Biomass of Alfalfa Canopies at .68μm

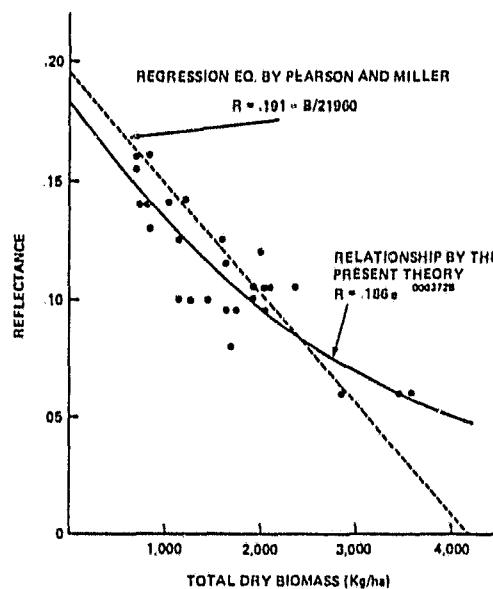


Fig. 3. Relationships of .68μm Reflectance vs. Total Dry Biomass for Shortgrass Prairie Canopy Data of Pearson and Miller (1972)

The values of the canopy reflectance characteristic parameters, however, varied for differing illumination conditions, and so did the KM parameter (Table 2). Their dependences on the sun angle seemed certain (Fig. 4).

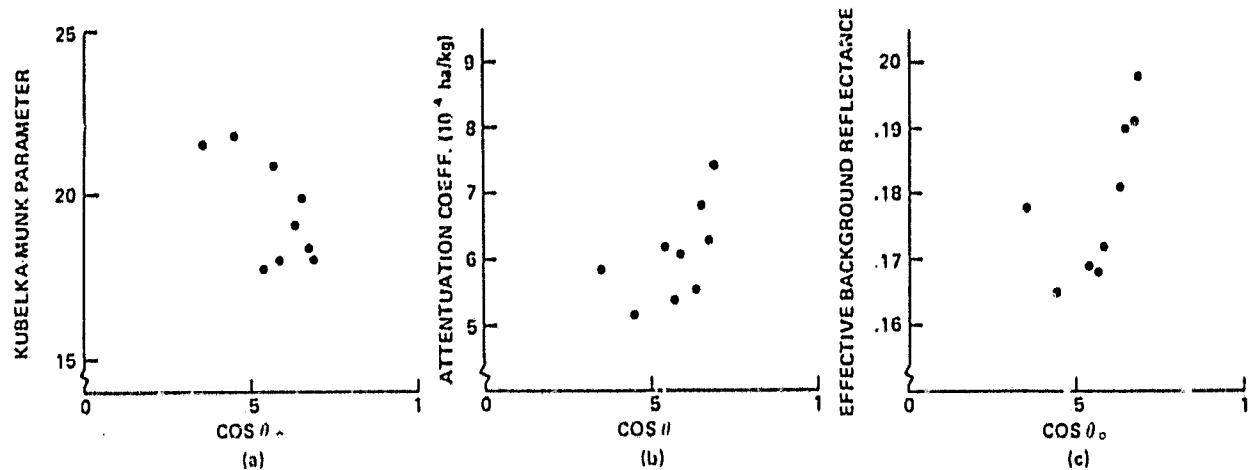


Fig. 4. Reflectance Characteristic Parameters of Alfalfa Canopy at  $.68\mu\text{m}$  as a Function of the Solar Zenith Angle ( $\theta_0$ ) under Clear Sky Conditions

Hence, bulk absorption and scattering coefficients of the vegetation canopy were a function of the insolation condition. The coefficient of variation of the ratio  $R_v/R_b$  was 20 - 30% less than those of  $R_v$  and  $R_b$  (Table 2), indicating that the ratios of the two reflectances,  $R_v$  and  $R_b$ , remained fairly constant while the two individual parameters changed considerably depending upon illumination conditions. Although the illumination conditions were not ideal for the diffuse reflectance model, they are realistic remote sensing conditions, and the diffuse reflectance relationships were shown to be useful for accurate estimation of alfalfa and shortgrass prairie biomass utilizing measurements of plant canopy reflectance.

#### SUMMARY

Useful relationships were formulated to describe variations of the diffuse spectral reflectance in terms of vegetation canopy variables, such as biomass. The relationships were based on the solution of the two-stream approximation of the radiative transfer equation. Out of the lengthy original expression of the diffuse reflectance formula, simple working equations

were derived by employing characteristic parameters, which are independent of the canopy coverage and identifiable by field observations. The typical asymptotic nature of reflectance data that is usually observed in biomass studies was clearly explained. It also established the range of expected apparent canopy reflectance values.

A procedure to estimate reflectance characteristic parameters was described for practical applications of the relationships. The simplified exponential formulas accurately depicted the observed relationships between biomass and spectral reflectance. They were shown to be useful for accurate estimation of alfalfa and shortgrass prairie biomass utilizing measurements of plant canopy reflectance.

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RELATIONSHIPS BETWEEN DIFFUSE REFLECTANCE AND VEGETATION CANOPY  
VARIABLES BASED ON THE RADIATIVE TRANSFER THEORY\*

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